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ABSTRACT: This article is a discussion of flows at large Knudsen numbers in the case where the collision cross-section of the molecules decreases with increasing relative velocity of the molecules. It is shown that in order to correlate wind-tunnel data with theoretical data, it is advisable to use a parameter α proposed by Sherman, Willis, and Maslach (1964). For Maxwellian molecules, this parameter takes the form $1/Kn_\infty$, while for the solid-sphere model, it is expressed by $Kn_{\text{wind-tunnel}} = Kn_{\text{real}}\sqrt{T_\infty/T_0}$.

There is presently very little information, both theoretical and experimental, concerning flows for high Knudsen numbers. /663*

Theoretical investigations are limited chiefly to the approximation of initial collisions for solid-spheres. A certain amount of calculations has also been conducted with the help of model equations.

During the comparison of the results of calculations with experimental data, errors, which are attributable to a limited knowledge of the laws of interaction of the molecules with the wall and among themselves, are possible, in addition to the errors of the method of calculations.

If a body is thermally insulated the energy dissipated by reflected molecules is equal to the energy carried in by arriving molecules and is independent of the coefficients of accommodation. If each point of the surface of the body is thermally insulated (the wall is absolutely a non-conductor of heat) and if the pulse of the reflected molecules and the temperature of the wall are uniquely related to the energy dissipated by the reflected molecules, it is then obvious that the resistance of each element of the body and distribution of temperatures on its surface are independent of the coefficients of accommodation. In the other limiting case of the body which is absolutely a conductor of heat, the resistance and temperature of the body are then independent of the coefficient of accommodation of energy if the average of the later is determined for the entire body.

Thus, by comparing the theoretical and experimental results for resistance and temperature of a thermally insulated body, we may ignore the coefficient of accommodation.

*Numbers in the margin indicate pagination in the foreign text.

The existing theoretical results, as was pointed out above, were obtained either for solid spherical molecules or with the help of model equations. It is therefore necessary to find the relationship between the properties of real molecules and the diameter of the spheres or between the parameters of the interaction of the molecules involved in the model equation. Let us recall that a model equation best approximates the Boltzmann equation for a Maxwellian (a more accurately pseudo-Maxwellian) gas.

The impacts of incoming and reflected molecules play the primary role in flows approximating free-molecular flows, particularly at hypersonic velocities. Consequently, the diameters of the solid-spheres or the other parameters of theoretical models of molecules should be selected in such a way that the nature of the interaction of the molecules during precisely these collisions is approximated in the best possible way. In the final analysis it is necessary to express both the experimental and the theoretical results through the parameters of the incoming flow. In converting from the parameters of the interaction of the molecules, which correspond to the incoming flow, to the parameters of the interaction taking place during the collision of the molecules it is necessary to consider that the conversion may differ for the mathematical model selected from the actual change in the interaction of real molecules. /664

Thus, for example, if a model of solid-spheres is used in the calculations, then, according to the very definition of the model, the collision cross-section is the very same in the incoming flow as during the impact of the incoming molecules with the reflected molecules.

In a real gas, however, the collision cross-sections are reduced with increasing relative velocity of the molecules. It is clear that comparable data can be obtained only in the case where the collision cross-section of model molecule-spheres is assumed to be equal to the actual cross-section during the impact of the reflected and incoming molecules, while the conversion to the parameters of the incoming flow is accomplished in both cases in accordance with the real law of change of molecular interaction. We should keep in mind here that the conversion for one gas to the parameters of the incoming flow may appear to be different under the conditions of a wind-tunnel experiment (particularly in a hypersonic wind-tunnel) and under natural conditions. The temperature of the incoming flow in aerodynamic wind-tunnels is often much lower for high Mach numbers than the temperature of the incoming flow under conditions of natural flight at the same Mach number. The relative velocities of the molecules in the incoming flow are accordingly much less under wind-tunnel conditions than under natural conditions. At lower relative velocities, however, the collision cross-section changes much more rapidly with a change in the relative velocity of colliding molecules than for high relative velocities. Consequently, for example, it may seen that the molecules behave under aerodynamic wind-tunnel conditions as Maxwellian molecules, while their cross-section changes but little under the conditions of natural flight and, consequently, their behavior can be satisfactorily approximated by molecule-spheres. Therefore, the calculation made for molecule-spheres will be in agreement for definite Mach and Knudsen numbers with the results of natural

studies for the very same Mach and Knudsen numbers, while the very same calculation corresponds to wind-tunnel tests with an incoming flow of a different Knudsen number. It is not difficult to see [1] that the wind-tunnel Knudsen number Kn_{w-t} is related in the given example to the Knudsen number of the calculation Kn_c by the relation:

$$Kn_{w-t} \sim \frac{Kn_c}{M}, \tag{1}$$

since the collision cross-section in the incoming flow in the wind-tunnel is M times greater (relative velocity of the order of the speed of sound a) than during the collision of reflected and incoming molecules (relative velocity of the order of V_∞). /665

The calculation for Maxwellian molecules, on the other hand, is comparable to the results of the wind-tunnel experiment for an incoming flow having identical Mach and Knudsen numbers. In comparing the calculation with the results of the natural experiment, however, the latter should be made for Knudsen number

$$Kn_{nat} \sim Kn_c M. \tag{2}$$

It should be noted that in converting from the wind-tunnel experiment to the natural experiment it is not permissible in the general case to compare the data of an incoming flow having identical and Mach and Knudsen numbers. The experiments for an incoming flow which were conducted on a different temperature level may be regarded essentially as experiments conducted in different gases, since the viscosity for low temperatures changes almost in proportion to temperatures, while at high temperatures it changes in proportion to the square root of the temperature. The wind-tunnel experiment in the selected example was conducted in a Maxwellian gas while the natural experiment was carried out in a gas consisting of molecule-spheres. Their data will be comparable if

$$Kn_{w-t} \sim \frac{Kn_{nat}}{M}. \tag{3}$$

Formulas (1-3) have, of course, only an approximate nature. A more accurate calculation is necessary for a critical comparison of the experimental data with the theoretical data. Thus the relative velocity of the molecules of the incoming flow was taken above as equal to the speed of sound $a = \sqrt{\kappa kT/m}$, while, more accurately, the mean relative velocity of the molecules is equal to $\xi_{rel} = 4\sqrt{kt_\infty/\pi m}$ for the Maxwellian distribution.

We may use, for example, the most probable velocity or the velocity of the molecules at which the pulse delivered by them is equal to the pulse

carried away by the molecules possessing the Maxwellian distribution as the mean velocity of the reflected molecules.

If the molecules are reflected diffusely with the Maxwellian distribution corresponding to temperature T_r the mean velocity determined by the first method will be

$$V_2 = \sqrt{\frac{2kT_r}{m}} = S_r,$$

while the mean velocity determined by the second method will be

$$V_2 = \sqrt{\frac{\pi kT_r}{2m}} = \frac{\sqrt{\pi}}{2} S_r.$$

When the incoming flow has high velocities and $T_r \ll T_0$, the differences between these determinations of mean velocity is slight. The real law of change of the collision cross-section can be found, for example, from the law of change of viscosity with temperature.

Let us consider an example of the comparison of experimental and theoretical data for a sphere. The value of $C_x - C_{x_{fm}}$ varies for a given mach number in proportion Kn_∞^{-1} independently of the law of the interaction of the molecules with the surface and among themselves ($C_{x_{fm}}$ is the free-molecular value). Figure 1 shows a comparison of the experimental results of [2] with the results of theories. Straight lines 1 and 2, which we borrowed from [3], were constructed for a Knudsen number at ∞ for each calculation model of molecules without conversion. Curve 1 was found from the theory of inital collisions for molecule-spheres [4] and curve 2 was found by using the modified model equation: /666

$$\frac{df}{dt} = A(n_i^2\Phi_{ii} + 2n_r n_i \Phi_{ri} + n_r^2\Phi_{rr}) - Anf. \qquad (4)$$

Here

$$\Phi_{kl} = \left(\frac{m}{2kmT_{kl}}\right)^{3/2} \exp\left\{-\frac{m}{2kT_{kl}}(\bar{\xi}-\bar{u}_{kl})^2\right\}, \quad k,l = r,i,$$

$$n_k = \int_{\Omega_k} f\,d\bar{\xi}, \quad \bar{u}_{kl} = \frac{1}{2n_k}\int_{\Omega_k} f\bar{\xi}\,d\bar{\xi} + \frac{1}{2n_l}\int_{\Omega_l} f\bar{\xi}\,d\bar{\xi},$$

$$T_{kl} = \frac{m}{6kn_k}\int_{\Omega_k} f(\bar{\xi}-\bar{u}_{kl})^2\,d\bar{\xi} + \frac{m}{6kn_l}\int_{\Omega_l} f(\bar{\xi}-\bar{u}_{kl})^2\,d\bar{\xi}.$$

The index r denotes the values corresponding to molecules (to those molecules leaving the body), the velocity of which are found to be within the solid angle Ω_r, from which the body is seen from a given point. Index i denotes the values corresponding to all other directions of the velocities of the molecules. This form of collision term enables us to consider more completely the interaction of the individual groups of molecules (reflected with incoming ir, incoming with incoming ii, and reflected with reflected rr). /667

All calculations were carried out with the assumption of diffuse reflection for the coefficient of accommodation $\alpha_1 = 1$. For the calculations made on the basis of the theory of initial collisions it was assumed that all reflected molecules have identical velocities. The calculations for the modified model equation were carried out by the iteration method. The coefficient A is expressed through viscosity.

$$A = kT_x/\mu(T_x) \tag{5}$$

The diameter of the molecules of the spheres may also be expressed through viscosity

$$\sigma = \pi d^2 = \left(\frac{mkT}{\pi}\right)^{1/2}\frac{1}{\mu(T_x)}. \tag{6}$$

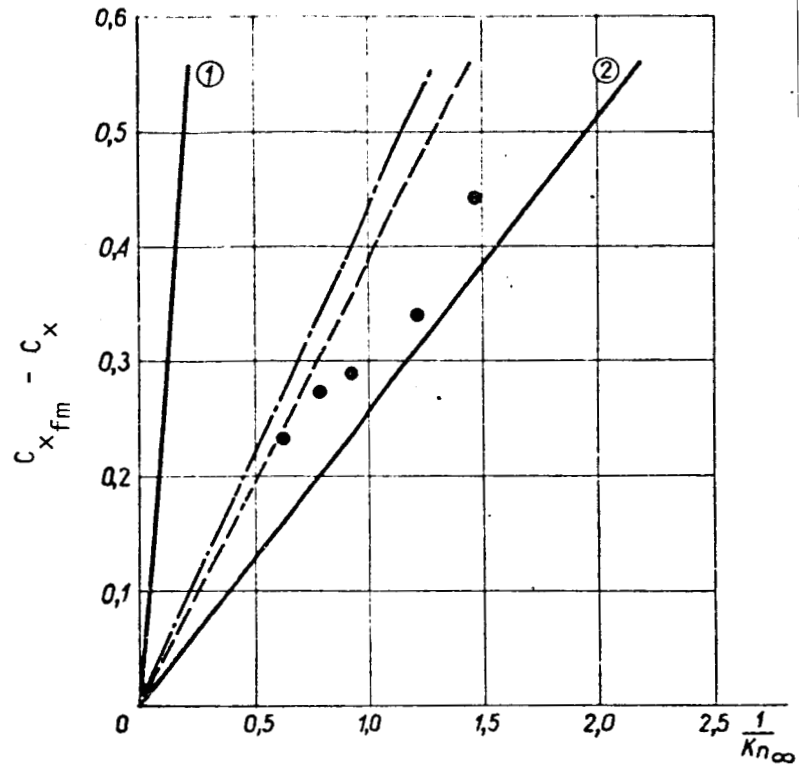

Figure 1

The Knudsen number Kn_∞, which is expressed through M and Re of the incoming flow is

$$Kn_\infty = \left(\frac{\kappa\pi}{2}\right)^{1/2} \frac{M_\infty}{Re_\infty}.$$

As seen in the figure, the calculation results constructed by this parameter for the model equation and by the theory of initial collisions for spheres diverge significantly. As was mentioned above, however, such a comparison is not valid, since we have compared not only the results of calculations carried out by two different methods, but for two different gases as well. For the purpose of making the data for the spheres comparable with the results obtained by using the model equation, i.e., for Maxwellian molecules, we have to construct them on the basis of a Knudsen number which changes in accordance with formula (1). In consideration of the above correction factors we may rewrite this formula in the following form:

$$Kn_{w-t} = \frac{Kn_c}{\frac{\sqrt{2\pi}}{4} S_\infty \left(1 + \frac{1}{S_r}\right)}, \quad \left(S_\infty = \sqrt{\frac{\kappa}{2}} M_\infty\right). \tag{1a}$$

Curve (1) thus recalculated is shown in Figure 1 by the broken line. Considering the approximate nature of these and other calculations, the conformate of the results of the calculations arrived at by the different methods can be regarded as satisfactory after the conversion. The fact that the experimental results are also now close to the calculated results shows that real molecules behave under the conditions of the given experiment similarly to Maxwellian molecules.

For the correlation of theoretical and experimental results in a viscose gas we often use the Reynolds number found on the basis of viscosity at the temperature which exists after a sudden change in compression or at the stagnation temperature. In terms of the kinetic theory this is equivalent to selecting the characteristic relative velocity of the molecules equal to the relative velocity of the molecules at the stagnation temperature $\bar{\xi}_{rel}(T_0)$. /668
The relative velocity of the molecules, as we have seen, is somewhat greater than V_∞ during the initial collisions. It is easy to see that the relative velocity of the molecules $\bar{\xi}_{rel}(T_0)$ is somewhat less than V_∞ for a monatomic gas, and even lower for a diatomic gas ($V/\bar{\xi}_{rel}(T_0) \sim 1/4\sqrt{2\pi k/(k-1)}$ at high Mach numbers). Thus, we clearly have, instead of formula (1a),

$$Kn_{w-t} = Kn_c \sqrt{\frac{T_\infty}{T_0}}. \tag{1b}$$

The data for solid-spheres recalculated by this formula are shown in Figure 1 by the dot-dash line.

Consequently, by using the Reynolds number constructed according to the parameters of stagnation in the correlation of data, we underestimate somewhat the relative velocities of the molecules during their initial collisions. With a reduction in the Knudsen number, however, the role of the collisions of the molecules of the directly incoming flow with the reflected molecules diminishes and the correlation of the data according to the stagnation temperature becomes more valid. Therefore, in a wide range of Knudsen numbers, the correlation of data can be accomplished, for example, according to the parameter (see [5]).

$$\alpha = \frac{1}{Kn_\infty}\left(\frac{T_0}{T_\infty}\right)\frac{\mu(T_\infty)}{\mu(T_0)}, \tag{7}$$

which changes to $1/K_{n_\infty}$ for Maxwellian molecules, but corresponds to formula (1b) for solid spheres.

Above we have correlated data for a cold body. If the body is "hot" or α_e is low, the velocity of the reflected molecules of the order V_∞ and, consequently, the relative velocity of the molecules during the initial collisions will be more than twice as great as the velocity $\bar{\xi}_{rel}(T_0)$. In this case the conversion for stagnation temperature [by formula (1b)] is much less valid than the conversion by (1a). This situation is often encountered in aerodynamic wind-tunnels, where the temperature of the body is of the order of the stagnation temperature, and even greater than the stagnation temperature in nearly free-molecular flows.

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